EXAM 07-11-2022

ATOMS AND MOLECULES. 18:15-20:15, # QUESTIONS: 5

YOU CAN MAKE USE (IF YOU THINK YOU NEED TO) OF THE FOLLOWING FORMULAS:

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$g_F = \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} g_J$$

1. ELECTRONIC, FINE, AND HYPERFINE STRUCTURE (20 POINTS)

^{223}Fr (Francium) has a single valence electron in the $7s$ shell.

A. (5 points) Consider the electronic ground state and the first excited electronic state. The light that is emitted as the excited state decays to the ground state consist of two components with wavelengths of 817.2 and 718.2 nm. Calculate the size of the spin-orbit coupling constant β.

B. (8 points) Now consider the nuclear spin of ^{223}Fr, $I = 3/2$. Make a schematic sketch of the energy level structure of the ground and the excited states, taking into account the spin-orbit splitting and the hyperfine structure. Assume that the hyperfine coupling constant A is positive. Label all electronic levels with their F quantum number, and give the term symbols for the electronic states. Also indicate the distances between the energy levels in units of A and β.

C. (7 points) ^{223}Fr has a nuclear magnetic dipole moment of $1.06\mu_N$. What will be the difference in the hyperfine structure of the transition from the first excited state to the ground state between this isotope and ^{212}Fr ($\mu = 4.59\mu_N$, $I = 5$)? Provide a sketch of this difference.

2. ZEEMAN EFFECT (23 POINTS)

The figure shows the hyperfine structure of the ground level ([Xe]$6s$) of ^{133}Cs, as a function of the magnetic flux density B.

A. (5 points) Deduce the nuclear spin of cesium. Explain your answer.

B. (8 points) What are the appropriate quantum numbers for the states in both strong and weak fields (mark these on a copy of the figure)? Indicate in this figure which states are low-field seeking and which states are high-field seeking.

C. (10 points) At the heart of an atomic clock, operating on a beam of Cs atoms, is a transition driven by an external microwave source close to 9.2 GHz between the hyperfine levels in the ground state. This transition is made between the $m_F = 0$ levels of the two hyperfine states. Why are these specific m_F levels used? Sketch and explain the experimental set-up of the original design of the Cs atomic clock (atomic beam technique).

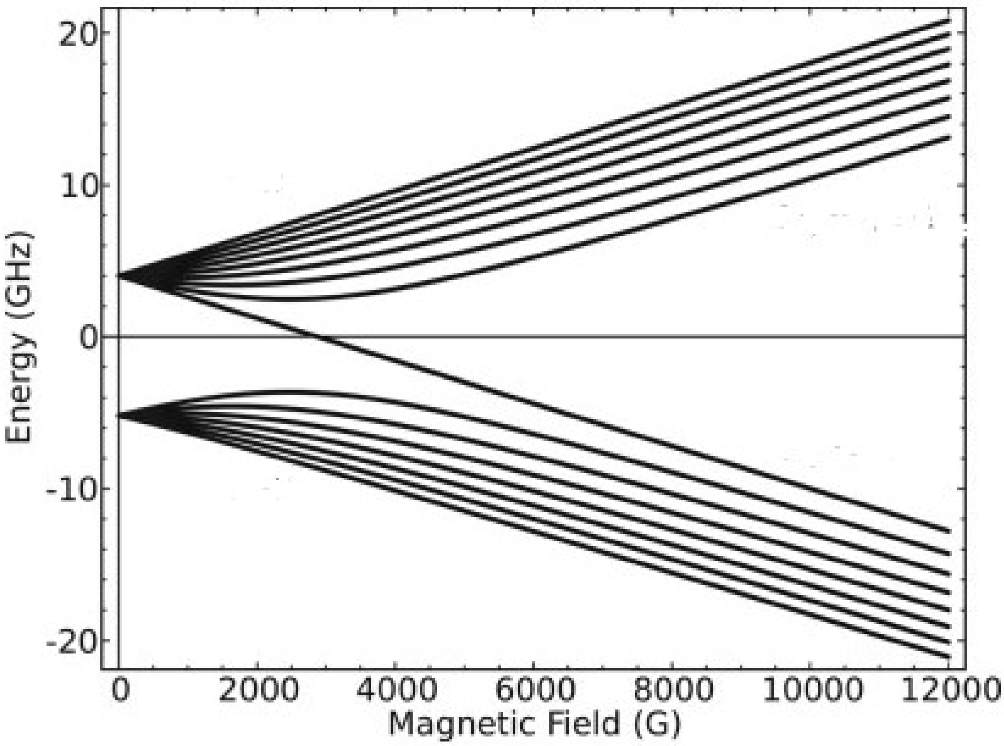

Fig. 2. Zeeman energy level pattern for Cs in its $6^2S_{1/2}$ ground electronic state.

938 Am. J. Phys., Vol. 81, No. 12, December 2013

3. Doppler-free spectroscopy (20 points)

A. (5 points) The 3s-3p transition in ^{23}Na atoms can be excited by light at a wavelength of 589 nm. The lifetime of the excited state is 16 ns. What is the natural linewidth of this transition?

B. (5 points) Calculate the Doppler width (full-width half max) of this transition at 300K.

C. (10 points) Explain how Doppler-free saturation spectroscopy works. Include a sketch of the typical set-up that is used.

4. Slowing and cooling with lasers (20 points)

A. (6 points) Show, starting from the optical Bloch equations (see below), that the maximum of the scattering force is given by $F_{max} = \hbar k \Gamma/2$. As a reminder, the fraction of the population in the excited state is given by $\rho_{22} = \frac{1-w}{2}$.

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \begin{pmatrix} \Omega\delta \\ \Omega\Gamma/2 \\ \delta^2 + \Gamma^2/4 \end{pmatrix}.$$

B. (10 points) Explain how the maximum scattering rate influences the design of a Zeeman slower. Also explain the shape of the magnetic field required for optimal deceleration efficiency. What determines the maximum velocity of the atoms that can still be captured and decelerated by a Zeeman slower?

C. (4 points) What makes an atomic transition suitable for laser slowing or cooling (give at least 2 criteria)?

5. Molecules (17 points)

A. (5 points) Write down the molecular Hamiltonian for the CaF molecule (the atomic number of Ca is 20, of F is 9). Explain the terms.

B. (5 points) Usually, we treat molecules within the Born-Oppenheimer approximation. What is the basis for this approximation? How is the molecular Hamiltonian partitioned?

C. (7 points) Make a schematic drawing of the first 3 rotational levels of a diatomic molecule in the vibrational ground state v"= 0, both within the rigid rotor model and including the centrifugal distortion. Indicate the spacing between the rotational levels in terms of the rotational constants B_e and D_e.